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Torsion, Wormholes, and the Problem of the Cosmological Constant

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We consider the effect of torsion in the early universe to see if it is possible to explain the small value (if not zero) of the cosmological constant at the present time. For the gauge-theoretic formulation of the Einstein–Cartan theory, we find a wormhole instanton solution which has a minimum (baby universe) radius of the Planck length. The basic difficulty with the wormhole approach is stressed. Finally, we give an explicit calculation from the expression for the evolution of the scale factor, which shows that the spin-dominated interaction term in the very early universe can cancel the cosmological constant term at that epoch.

1. INTRODUCTION

The cosmological constant problem has been receiving considerable attention in recent years (Weinberg, 1989). The essential problem is to explain why the effective cosmological constant is so small, if not zero, at the present epoch, vastly less than the values one would expect from the large changes in the vacuum energy in the early universe resulting from the breaking of some symmetry group. If the symmetry breaking takes place at some energy E, then the induced vacuum energy density is $\rho_{\text{vac}} \sim E^4$, effectively acting like a cosmological constant term of the form

$$\rho_{\text{vac}} = \Lambda_{\text{eff}} c^4/8\pi G$$

For instance, at the Planck epoch, $t \approx 10^{-43}$ sec, $E \approx 10^{19}$ Gev, one would expect $\rho_{\text{vac}} \approx 10^{114}$ ergs cm^{-3}, corresponding to an effective cosmological constant $\Lambda_{\text{Pl}} \cong 10^{66}$ cm^{-2} arising from quantum gravitational contributions to the vacuum energy. Again, the GUT phase transition at energies $\approx 10^{15}$ Gev would induce another large $\Lambda_{\text{GUT}} \approx 10^{50}$ cm^{-2}. There would also

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be large contributions from other symmetry-breaking phase transitions, such as the electroweak scale, at somewhat later epochs in the early universe. The question is what has happened to all these large contributions to the Λ term. Why is the present epoch value of Λ vanishingly small?

In a recent review, Weinberg summarizes five different approaches undertaken in recent years to understand this question. He considers supersymmetry (exact global supersymmetry would indeed make the vacuum energy and hence Λ vanish). But we know that supersymmetry must be broken quite strongly and this would give a large contribution to Λ which would not vanish. There is no known symmetry principle (like gauge invariance in electromagnetism implying zero photon mass) which would make Λ vanish identically and Weinberg states that it is very hard to see how any property of supergravity or superstrings could make the effective Λ sufficiently small.

He also resorts to the anthropic principle to explain the smallness of Λ, but this is a rather weak argument. Again, many attempts have involved some sort of adjustment mechanism (Dolgov, 1982; Wilczek, 1983) requiring some extra scalar field which evolves and acts as a counterterm to cancel the cosmological term. However, it turns out that the scalar field must have some special ad hoc properties and "fine tuning" is involved at all stages.

Recently there has been a lot of excitement about a new mechanism suggested by Coleman (1988) which follows up an earlier work of Hawking which described how in quantum cosmology there could arise a distribution of values for the effective cosmological constant with an enormous peak at $\Lambda_{\text{eff}} = 0$. Coleman considers the effect of topological fixture known as wormholes, consisting of two asymptotically flat spaces joined together at a 3-surface and shows that the probability distribution or expectation value has an infinite peak at $\Lambda_{\text{eff}} \Rightarrow 0$. However, many objections to this have been raised, which we shall briefly consider later.

One promising possibility which has not been considered so far in understanding the cosmological constant problem is the use of torsion in a framework such as Einstein–Cartan (EC) theory, which is natural in considering the gravitational contributions of particles with spin, which is indeed a universal property of elementary particles. In fact, at sufficiently early epochs the energy content of the universe can indeed be spin dominated and the temporal evolution of the spin-density tensor is important in describing the cosmological dynamics, as we shall see in Section 3.

2. TORSION AND WORMHOLES

We shall first indicate how the antisymmetric field strength for torsion can give rise to instanton wormhole solutions. In the framework of an

$SL(2, C)$ gauge theory, the Einstein–Cartan action can be expressed as (Sivaram and Sinha, 1975, 1979; Carmeli, 1984; Bandyopadhyay, 1990)

$$L = -\frac{1}{16\pi G} R + \frac{4}{8\pi G} S_{\alpha\beta\gamma} S^{\alpha\beta\gamma} \tag{1}$$

where the antisymmetric field strength $S_{\alpha\beta\gamma}$ is [related to the conserved current closing on the $SL(2, C)$ algebra as $J^\mu = J^\mu + (1/16\pi G)\varepsilon^{\mu\alpha\beta\gamma} S_{\alpha\beta\gamma}$]

$$S_{\alpha\beta\gamma} = c_\alpha \times f_{\beta\gamma} \qquad \text{and} \qquad f_{\beta\gamma} \cdot \bar{g} = F_{\beta\gamma}$$

where $\bar{g} \equiv (g_1, g_2, g_3)$ are tangent vectors to the generators of the $SL(2, C)$ gauge group. $F_{\beta\gamma}$ in terms of the gauge fields A_β has the usual structure $F_{\beta\gamma} = \partial_\beta A_\gamma - \partial_\gamma A_\beta + [A_\beta, A_\gamma]$.

In the absence of matter, but with only torsion (analogous to neutrinos with spin but no mass), and with the usual Euclidean space-time metric ansatz $[ds^2 = dr^2 + a^2(\tau)\, d^2\Omega_3,\ d^2\Omega_3$ being the metric of $S^3]$, the field equations reduce to

$$\left(\frac{da}{d\tau}\right)^2 = \left(1 - \frac{r_{\min}^4}{a^4}\right) \tag{2}$$

where $r_{\min}^4 = 3G^2S^2/c^4$, S being the spin scalar. With S identified as the basic unit $\hbar$ of angular momentum, the minimum radius has the value of the Planck length and is not arbitrary as in the case of solutions involving other antisymmetric field strengths with random coupling [for instance, we have found that in the case of black hole evaporation with torsion effects (de Sabbata *et al.*, 1990) we are led at a final stage to a remnant mini black-hole with angular momentum of $\hbar$], so that we find the Coleman result without any arbitrary constants. The time $\tau = 0$ is chosen when the radius attains a minimum value. We have obtained a solution which describes tunneling from one E space-time to another via a baby universe of radius $r_{\min}$ with a tunneling amplitude $\psi \sim k \exp(-S_{\min}/\hbar)$, where

$$S_{\min} = \hbar(3\pi^2/4)(r_{\min}/l_{\rm Pl}) \tag{3}$$

With an effective cosmological constant $\Lambda(\alpha)$,

$$\Lambda(\alpha) = \Lambda_0 - \alpha k \exp(-S_{\min}/\hbar)$$

we can arrive at Coleman's expression for the wave function of the universe

$$\psi = \int_{-\infty}^{\infty} \frac{d\alpha}{(\pi)^{1/2}} \langle \exp(V_{\min}\alpha)\rangle^\beta Z(\alpha)$$

where V is the volume of the 3-space and the weight $Z(\alpha)$ is given by

$$Z(\alpha) = \exp\left(\frac{-\alpha^2}{2}\right) \exp\left(k \exp \frac{3}{8G^2\Lambda(\alpha)}\right) \tag{4}$$

having an extremely sharp peak at $\Lambda(\alpha) = 0$.

Apart from the usual criticism of this approach, such as the use of the Euclidean metric (it is essential that the path integral be given by a stationary point of the Euclidean action) and neglect of phases, it may be pointed out that any 3-index gauge field can mimic a cosmological term. For instance, for an action such as

$$(1/16\pi G)\int d^4x\,(-g)^{1/2}(R+2\lambda)-\int d^4x\,(1/48)(-g)^{1/2}A^{\mu\nu\rho\sigma}A_{\mu\nu\rho\sigma} \quad (5)$$

where $A_{\mu\nu\rho\sigma}=\partial[_\mu H_{\nu\rho\sigma}]$, substituting the solution $(-g)^{1/2}A^{\mu\nu\rho\sigma}=k\varepsilon^{\mu\nu\rho\sigma}$ (following from $D_\mu A^{\mu\nu\rho\sigma}=0$) into the corresponding energy-momentum tensor $T^{\mu\nu}(H)$ for the H field, as resulting from the field equations corresponding to (5), we get

$$T^{\mu\nu}(H)=-(1/2)k^2g^{\mu\nu}$$

i.e., an effective Λ term!

However, it must be noted that substituting an ansatz into the action and varying that action does not yield the same result as substituting the ansatz into the field equations (Duff, 1989)! This seems to be a basic difficulty with wormhole-type approaches. In the next section we shall consider a specific example to demonstrate the possibility of canceling the cosmological term with a torsion term in the early universe (de Sabbata and Sivaram 1989, 1990).

3. CAN TORSION CANCEL THE COSMOLOGICAL CONSTANT TERM IN THE EARLY UNIVERSE?

The simplest EC generalization of the standard big-bang cosmology is obtained by considering a universe filled with unpolarized spinning fluid (Hehl *et al.*, 1976; de Sabbata and Gasperini, 1985). For the fluid and spin parts of the energy-momentum tensor we have

$$\langle T_F^{\alpha\beta}\rangle=(\rho+p)u^\alpha u^\beta-pg^{\alpha\beta} \quad (6)$$

and

$$\langle T^{\alpha\beta}\rangle=-\tfrac{1}{2}\chi^{\sigma^2u^\alpha u^\beta}+\tfrac{1}{4}\chi\sigma^2g^{\alpha\beta} \quad (7)$$

where $\sigma^2=(1/2)\langle S_{\alpha\beta}S^{\alpha\beta}\rangle$, and $S_{\alpha\beta}$ is the spin density.

We can now solve for the Einstein equations

$$G^{\alpha\beta}(\{\cdot\})=\chi\vartheta^{\alpha\beta}$$

where

$$\vartheta^{\alpha\beta}=\langle T^{\alpha\beta}\rangle+\langle\tau^{\alpha\beta}\rangle=(\rho+p-\tfrac{1}{2}\chi\sigma^2)u^\alpha u^\beta-(p-\tfrac{1}{4}\chi\sigma^2)g^{\alpha\beta} \quad (8)$$

where p, ρ, and σ depend only on time. In the comoving frame $u^\mu \equiv (0, 0, 0, 1)$, we get the following modified field equations of the Robertson-Walker universe, which in general for $k \neq 0$ and $\Lambda \neq 0$ is of the form

$$\dot{R}^2/R^2 = (8\pi G/3)(\rho - \tfrac{2}{3}\pi G\sigma^2/c^4) + \Lambda c^2/3 - kc^2/R^2 \tag{9}$$

We immediately notice that the torsion term in equation (9) (the second term within parentheses) is of *opposite* sign to that of the cosmological constant term. This raises the possibility that a sufficiently large spin-torsion term in the early universe might cancel a correspondingly large cosmological constant. We shall see that this is indeed the case. For instance, consider the universe at the Planck epoch, when, as noted earlier, the Λ term was $\approx 10^{66}$ cm^{-2}, implying $\Lambda_{\text{Pl}}c^2 \approx 10^{87}$ in equation (9). At $t_{\text{Pl}} \cong 10^{-43}$ sec, the universe has a density of $\approx c^5/G^2\hbar \approx \rho_{\text{Pl}} \approx 10^{93}$ g cm^{-3}, and as the particle masses were $\approx 10^{19}$ Gev $\approx 10^{-5}$ g, the particle number density was $n_{\text{Pl}} \approx 10^{98}$ cm^{-3}, so that σ, the spin density, was $\sigma_{\text{Pl}} \approx 10^{98} \cdot 10^{-27}$ (i.e., $n_{\text{Pl}} \cdot \hbar$) $\approx 10^{71}$. This gives for the term $-(8\pi G/3)(2\pi/3)G\sigma_{\text{Pl}}/c^4$ [i.e., the torsion term in equation (9)] the value of $\approx -10^{87}$, which is exactly equal and of opposite sign to that of the cosmological term, $\Lambda_{\text{Pl}}c^2 \approx +10^{87}$, so that the two terms would have canceled each other in the early universe at the Planck epoch.

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